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REACHABLE SET AT A CERTAIN TIME FOR A DUBINS CAR IN THE CASE OF A ONE-SIDED TURN

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We study a three-dimensional reachable set "at a time" for a nonlinear control system often called a Dubins car. The controlled object (a car) moves in a plane with a constant linear velocity and bounded turning radius. The case where the car can turn left and right was studied earlier. In this paper, we investigate the case where the car can turn only in one direction. In the case where the constraints imposed on the control permit a straight line motion, we prove that the system can be guided to any point of the boundary of the reachable set by means of a piecewise-constant control with at most two switchings. Moreover, two-dimensional sections of the reachable set with constant angular coordinate are convex. If the constraints on the control forbid a straight line motion (which means that the car is turning at each time and the turning radius is chosen within prescribed limits), then the number of switchings of a piecewise-constant control guiding the system to the boundary of the reachable set grows with the growth of the time for which the reachable set is constructed. We consider in detail the case where this time is not greater than the time needed for a 2π turn with the smallest possible turning radius. In this case, any piecewise-constant control guiding the system to the boundary has at most two switchings, and the sections of the reachable set with constant angular coordinate are strictly convex.

Keywords: Dubins car, one-sided turn, three-dimensional reachable set, Pontryagin maximum principle, piecewise-constant control, convexity of sections of a reachable set.

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