TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 24 No. 1 **2018**

MSC: 34H05, 34K10, 34K34, 34K35

DOI: 10.21538/0134-4889-2018-24-1-131-142

ON A CLASS OF OPTIMAL CONTROL PROBLEMS FOR FUNCTIONAL DIFFERENTIAL SYSTEMS

V. P. Maksimov

A linear functional differential control system of general form with aftereffect is considered. An optimal control problem with linear constraints on the state and control variables is studied. The control is realized by a linear operator of general form. The cases of distributed and lumped delay in the control loop, as well as the case of impulsive control, are covered. The Cauchy matrix is used to reduce the problem under consideration to a problem formulated only in terms of control variables with the use of some auxiliary variables linked with the defining relations for the Cauchy matrix of the system. In the case when the control is chosen from a finite-dimensional subspace of the control space, a problem effectively solvable by standard software tools is written explicitly. An example of an applied optimal control problem that arises in economic dynamics is presented. A class of hybrid systems (systems with continuous and discrete times) reducible to the system under consideration is described.

Keywords: linear systems, control, optimization.

REFERENCES

1. Gabasov R.F., Kirillova F.M. *Printsip maksimuma v teorii optimal'nogo upravleniya* [The maximum principle in optimal control theory]. Moscow, Editorial URSS, 2011, 272 p. ISBN: 978-5-397-01746-6 .
2. Kolmanovskiĭ V.B., Schaĭkhet L.E. *Control of systems with aftereffect*. N Y: American Mathematical Society, 1996, Ser. Translations of mathematical monographs, vol. 157, 336 p. ISBN: 0-8218-0374-3 .
3. Shevchenko G.V. A numerical method to minimize resource consumption by linear systems with constant delay. *Autom. Remote Control*, 2014, vol. 75, no. 10, pp. 1732–1742. doi: 10.1134/S0005117914100026 .
4. Shevchenko G.V. Computational solution of time-optimal control problem for linear systems with delay. *Vestn. Udmurtsk. Univ. Mat. Mekh. Komp. Nauki*, 2012, no. 2, pp. 100–105 (in Russian).
5. Gabasov R., Kirillova F.M., Pavlenok N.S. Optimal discrete impulse control of linear systems. *Autom. Remote Control*, 2008, vol. 69, no. 3, pp. 443–462. doi: 10.1134/S0005117908030107 .
6. Korotkii D.A. Solution of the optimal control problem with delay. *Vestn. Udmurtsk. Univ. Mat. Mekh. Komp. Nauki*, 2008, no. 2, pp. 61–62 (in Russian).
7. Gabasov R., Grushevich O.P., Kirillova F.M. Optimal control of linear systems with delay taking into account terminal constraints on their states. *Autom. Remote Control*, 2007, vol. 68, no. 12, pp. 2097–2112. doi: 10.1134/S0005117907120016 .
8. Maksimov V.P. Some questions of the control theory for functional differential systems. *Izv. IMI UdGU*, 2015, vol. 46, no. 2, pp. 112–119 (in Russian).
9. Azbelev N., Maksimov V., Rakhmatullina L. *Introduction to the theory of linear functional differential equations*. Atlanta, GA: World Federation Publishers Company, 1995, Advanced Series in Mathematical Science and Engineering, 3, 172 p. ISBN: 1-885978-02-2 . Original Russian text published in Azbelev N., Maksimov V., Rakhmatullina L. *Vvedenie v teoriyu funktsional'no-differentsial'nykh uravnenii*, Moscow, Nauka Publ., 1991, 280 p.
10. Azbelev N.V., Maksimov V.P., Rakhmatullina L.F. *Elementy sovremennoi teorii funktsional'no-differentsial'nykh uravnenii. Metody i prilozheniya* [Elements of the contemporary theory of functional differential equations. Methods and applications]. Moscow, Institute of Computer Science, 2002, 384 p. ISBN: 5-93972-112-5 .

11. Azbelev N.V., Maksimov V.P., Rakhmatullina L.F. *Introduction to the theory of functional differential equations: methods and applications*. N Y, Cairo, Hindawi Publishing Corporation, 2007, 314 p. ISBN: 977-5945-49-6/hbk .
12. Anokhin A.V. On linear impulse systems for functional-differential equations. *Soviet Mathematics. Doklady*, 1986, vol. 33, no. 1, pp. 220–223.
13. Kurzweil Ja. Generalized ordinary differential equations and continuous dependence on a parameter. *Czechoslovak Math. J.*, 1957, vol. 7(82), no. 3, pp. 418–449.
14. Zavalishin S.T., Sesekin A.N. *Dynamic impulse systems: theory and applications*. Dordrecht, Kluwer Academic Publishers, 1997, 256 p. ISBN: 0-7923-4394-8 . Original Russian text published in Zavalishin S.T., Sesekin A.N. *Impul'snye protsessy. Modeli i prilozheniya.* Moscow, Nauka Publ., 1991, 256 p.
15. Schwabik Š. *Generalized ordinary differential equations*. Singapore, World Scientific, 1992, 392 p. ISBN: 978-981-4505-04-8 .
16. Ashordia M. On the stability of solutions of the multipoint boundary value problem for the system of generalized ordinary differential equations. *Mem. Differential Equations Math. Phys.*, 1995, vol. 6, pp. 1–57.
17. Maksimov V.P., Rumyantsev A.N. Boundary value problems and problems of pulse control in economic dynamics. Constructive study. *Russian Mathematics* (*Izv. VUZ*), 1993, vol. 37, no. 5, pp. 48–62.
18. Maksimov V.P. The Cauchy formula for a functional-differential equation. *Differential Equations*, 1977, vol. 13, no. 4, pp. 405–409.
19. Maksimov V.P. *Voprosy obshchei teorii funktsional'no-differentsial'nykh uravnenii* [Questions of the general theory of functional differential equations]. Perm, Perm State University Publ., 2003, 306 p.
20. Maksimov V.P. A variant of the maximum principle for linear systems with aftereffect. *J. Tambov Univ. Rep. Ser. Nat. Tech. Sci.*, 2015, vol. 20, no. 5, pp. 1284–1286 (in Russian).
21. Maksimov V.P., Chadov A.L. Hybrid models in economic dynamics models. *Perm University Herald. Economy*, 2011, no. 2, pp. 13–23 (in Russian).
22. Chadov A.L., Maksimov V.P. Linear boundary value problems and control problems for a class of functional differential equations with continuous and discrete times. *Funct. Differ. Equ.*, 2012, vol. 19, no. 1-2, pp. 49–62.
23. Andrianov D.L. Boundary value problems and control problems for linear difference systems with aftereffect, *Russian Mathematics* (*Izv. VUZ*), 1993, vol. 37, no. 5. pp. 1–12.

The paper was received by the Editorial Office on August 28, 2017.

Vladimir Petrovich Maksimov, Dr. Phys.-Math. Sci., Prof., Perm State University, Perm, 614990 Russia, e-mail: maksimov@econ.psu.ru .

Cite this article as:
V. P. Maksimov. On a class of optimal control problems for functional differential systems, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 1, pp. 143–156 .