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SOLVABILITY OF A MIXED BOUNDARY VALUE PROBLEM FOR A STATIONARY REACTION–CONVECTION–DIFFUSION MODEL

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We study the solvability of an inhomogeneous mixed boundary value problem for a stationary reaction–convection–diffusion model. Such models are often used in science and engineering for the description and analysis of various processes of heat and mass transfer. We focus on the issues of solvability of the boundary value problem in various functional spaces and on the stability of the solution and its continuous dependence on the input data in natural metrics. The peculiarity of the problem consists in the inhomogeneity and irregularity of the mixed boundary data. These boundary data, in general, cannot be continued inside the domain so that the continuation is sufficiently smooth and can be used in the known way to transform the problem to homogeneous boundary data. To prove the solvability of the problem, we use the Lax–Milgram theorem. Estimates for the norms of the solution follow from the same theorem. The properties of the solution of the direct problem found in this study will be used in what follows to solve inverse problems.

Keywords: direct problem, mixed boundary condition, weak solution, generalized solution, strong solution, stability, completely continuous operator.

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