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BRIESKORN MANIFOLDS, GENERALIZED SIERADSKI GROUPS, AND COVERINGS OF LENS SPACE

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The Brieskorn manifold $\mathscr{B}(p,q,r)$ is the r-fold cyclic covering of the three-dimensional sphere S^3 branched over the torus knot $T(p,q)$. The generalised Sieradski groups $S(m,p,q)$ are groups with m-cyclic presentation $G_m(w)$, where the word w has a special form depending on p and q. In particular, $S(m,3,2) = G_m(w)$ is the group with m generators $x_1,\dots,x_m$ and m defining relations $w(x_i,x_{i+1},x_{i+2}) = 1$, where $w(x_i,x_{i+1},x_{i+2}) = x_i x_{i+2} x_{i+1}^{-1}$. Cyclic presentations of $S(2n,3,2)$ in the form $G_n(w)$ were investigated by Howie and Williams, who showed that the n-cyclic presentations are geometric, i.e., correspond to the spines of closed three-dimensional manifolds. We establish an analogous result for the groups $S(2n,5,2)$. It is shown that in both cases the manifolds are n-fold branched cyclic coverings of lens spaces. For the classification of the constructed manifolds, we use Matveev's computer program "Recognizer."

Keywords: three-dimensional manifold, Brieskorn manifold, cyclically presented group, Sieradski group, lens space, branched covering.

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