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ON THE COMMUTATOR SUBGROUPS OF FINITE 2-GROUPS GENERATED BY INVOLUTIONS

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For a finite group G we denote by $d(G)$ the minimum number of its generators and by G' the commutator group of G. Ustyuzhaninov published without proof the list of finite 2-groups generated by three involutions with elementary abelian commutator subgroup. In particular, $d(G') \leq 5$ for such a group G. Continuing this research, we pose the problem of classifying all finite 2-groups generated by n involutions (for any $n \geq 2$) with elementary abelian commutator subgroup. For a finite 2-group G generated by n involutions with $d(G) = n$, we prove that

$$d(G') \leq \binom{n}{2} + 2\binom{n}{3} + \cdots + (n-1)\binom{n}{n}$$

for any $n \geq 2$ and that the upper bound is attainable. In the first section we establish the inequality for $d(G')$, and in the second section we construct for any $n \geq 2$ a finite 2-group generated by n involutions with elementary abelian commutator subgroup of rank

$$\binom{n}{2} + 2\binom{n}{3} + \cdots + (n-1)\binom{n}{n}.$$

The method of constructing this group G is similar to the method used by the author in a number of papers for the construction of Alperin's finite groups. Using the known theorem on cyclic extensions, we obtain G as the consecutive semidirect product of groups of order 2. In the end of the paper, we give an example of an infinite 2-group generated by involutions with infinite elementary abelian commutator; the example is obtained from the constructed finite 2-groups.

Keywords: 2-group generated by involutions, commutator subgroup.

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