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SIMPLE ARCS IN PLANE CURVES AND KNOT DIAGRAMS

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We study simple arcs in plane curves and in minimal diagrams of classical knots. The main results of the paper are as follows. (1) In each minimal diagram of an arbitrary knot K , there exists a simple arc passing through $\min\{\text{cr}(K), 6\}$ crossings, where $\text{cr}(K)$ denotes the crossing number of K . (2) For any knot K except for the four simple knots 8_{16} , 8_{18} , 9_{40} , and 10_{120} in the notation of the Rolfsen table, there is a minimal diagram with a simple arc passing through $\min\{\text{cr}(K), 8\}$ crossings. The first claim is proved using the techniques of combinatorics on words. We introduce a new language for plane curves and their chord diagrams; the symbols of this language correspond to the lengths of the chords. As a result, the statement is reduced to a question in the theory of completeness and avoidability of sets of forbidden patterns: we describe a set of forbidden patterns and prove that the language with no words containing forbidden patterns is finite. To prove the second claim, methods of algorithmic topology are used: the statement reduces to a brute-force search for curves of a special type and then a computer algorithm is described that performs the search; we present the results of its operation.

Keywords: knot, minimal knot diagram, crossing number, flype, plane curve, combinatorics on words, algorithmic topology.

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