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FINITE SIMPLE GROUPS WITH FOUR CONJUGACY CLASSES OF MAXIMAL SUBGROUPS. I

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We study the finite simple groups with exactly four conjugacy classes of maximal subgroups. The groups with this property are called $4M$ -groups. We prove two theorems. Theorem 1 gives a complete list of finite simple $4M$ -groups, which contains some linear and unitary groups as well Suzuki groups over the field of order 2^r , where r is a prime ($r > 2$). In Theorem 2 we describe finite nonsolvable $4M$ -groups without normal maximal subgroups. Thus, the paper gives a description of finite nonsolvable $4M$ -groups that coincide with their commutator group. This study uses the author's earlier results on the structure of finite groups with exactly three conjugacy classes of maximal subgroups and Pazderski's results on the structure of finite groups with exactly two conjugacy classes of maximal subgroups.

Keywords: finite group, simple group, maximal subgroup.

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