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KNOT GROUPS AND NILPOTENT APPROXOMABILITY

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We study groups of classical links, welded links, and virtual links. For classical braids, it is proved that a braid and its automorphic image are weakly equivalent. This implies the affirmative answer to the question of the coincidence of the groups constructed from a braid and from its automorphic image. We also study the problem of approximability of groups of virtual knots by nilpotent groups. It is known that in a classical knot group the commutator subgroup coincides with the third term of the lower central series, and hence the factorization by the terms of the lower central series yields nothing. We prove that the situation is different for virtual knots. A nontrivial homomorphism of the virtual trefoil group to a nilpotent group of class 4 is constructed. We use the Magnus representation of a free group by power series to construct a homomorphism of the virtual trefoil group to a finite-dimensional algebra. This produces the nontrivial linear representation of the virtual trefoil group by unitriangular matrices of order 8.

Keywords: virtual knots, links, groups.

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