

MSC: 16S34

DOI: 10.21538/0134-4889-2017-23-4-32-42

**DESCRIPTION OF THE UNIT GROUP OF THE INTEGRAL  
GROUP RING OF A CYCLIC GROUP OF ORDER 16****R. Zh. Alev, O. V. Mitina, T. A. Khanenko**

The paper is devoted to the description of the group of units of the integral group ring of a cyclic group of order 16. The groups of units of the integral group rings of cyclic groups of orders 2 and 4 are trivial, and the group of units of the integral group ring of a cyclic group of order 8 is well known. Thus, the case of a cyclic group of order 16 is the first for which the structure of the group of units of the integral group ring of a cyclic 2-group has not been studied completely. When the groups of units of the integral group rings of cyclic 2-groups of orders greater than 16 are studied, it is necessary to have information on the structure of the groups of units of the integral group rings of cyclic 2-groups of lower orders, in particular, of order 16. Thus, we can say that the case of the group of order 16 is the basis for further research. We describe the group of units of the integral group ring of a cyclic group of order 16 in terms of local units defined by the characters of a cyclic group of order 16 and by the units of the ring of integers of the cyclotomic field  $\mathbf{Q}_{16}$  obtained by adjoining a primitive root of unity of degree 16 to the field of rational numbers. That is why we study in detail the structure of the group of units of the ring of integers of the cyclotomic field  $\mathbf{Q}_{16}$ . In addition, we derive important relations between the coefficients of an arbitrary unit of the integral group ring of a cyclic group of order 16. These relations will obviously serve as patterns and examples for obtaining similar relations in studying the units for the cases of 2-groups of orders greater than 16. Finally, we note that one of the generators of the group of units of the integral group ring of a cyclic group of order 16 is a singular unit defined by two units of the ring of integers of the cyclotomic field  $\mathbf{Q}_{16}$ . This unit is the product of the two local units, each of which is not contained in the integral group ring of a cyclic group of order 16.

Keywords: cyclic group, group ring, unit of a group ring, cyclotomic field, ring of integers of a field, unit of the ring of integers of a cyclotomic field, integral group ring.

**REFERENCES**

1. Alev R.Zh., Mitina O.V., Khistenko E.A. Congruence modulo 2 of circular units in the fields  $Q_{16}$  and  $Q_{32}$ . *Chelyab. Fiz.-Mat. Zh.*, 2016, vol. 1, no. 4, pp. 8–29 (in Russian).
2. Alev R.Zh., Mitina O.V., Khanenko T.A. Finding of units for integer group rings of orders 16 and 32 cyclic groups. *Chelyab. Fiz.-Mat. Zh.*, 2016, vol. 1, no. 4, pp. 30–55 (in Russian).
3. Curtis C.W., Reiner I. *Representation theory of finite groups and associative algebras*. Ser. Pure Appl. Math., vol. XI. N. Y.; London, Interscience Publishers, a division of John Wiley & Sons, 689 p. ISBN: 978-0-8218-4066-5. Translated to Russian under the title *Teoriya predstavlenii konechnykh grupp i assotsiativnykh algebr*. Moscow, Nauka Publ., 1969, 668 p.
4. Alev R.Zh. Central elements of integral group rings. *Algebra and Logic*, 2000, vol. 39, no. 5, pp. 293–300. doi: 10.1007/BF02681613.
5. Alev R. Zh. Units of character fields and central units of integral group rings of finite groups. *Siberian Advances of Mathematics*, 2001, vol. 11, no. 1, pp. 1–33.

*Rifkhat Zhalyalovich Alev*, Dr. Phys.-Math. Sci., Prof., South Ural State University, Chelyabinsk State University, Chelyabinsk, 454080 Russia, e-mail: alevrz@susu.ru, alev@csu.ru.

*Ol'ga Viktorovna Mitina*, Cand. Sci. (Phys.-Math.), South Ural State University, Chelyabinsk State University, Chelyabinsk, 454080 Russia, e-mail: ovm@csu.ru.

*Tat'yana Aleksandrovna Khanenko*, student Chelyabinsk State University, Chelyabinsk, 454080 Russia, e-mail: tanja\_1110\_94@mail.ru.

Cite this article as:

R. Zh. Alev, O. V. Mitina, T. A. Khanenko, Description of the unit group of the integral group ring of a cyclic group of order 16, *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 4, pp. 32–42.