TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 23 No. 4 **2017**

MSC: 06A06, 54H25, 54E40

DOI: 10.21538/0134-4889-2017-23-4-292-300

PRESERVATION OF THE EXISTENCE OF COINCIDENCE POINTS UNDER SOME DISCRETE TRANSFORMATIONS OF A PAIR OF MAPPINGS OF METRIC SPACES

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In topology there are known results on the preservation under homotopy of the fixed point property of self-mappings in some spaces if the Lefschetz number of the initial mapping is nonzero. For the class of contracting mappings of metric spaces and for some of their generalizations, there are M. Frigon's known results on the preservation of the contraction property and hence of the fixed point property under homotopies of some special type. In 1984 J. W. Walker introduced a discrete counterpart of homotopy for mappings in an ordered set, which he called an order isotone homotopy. R. E. Stong showed the naturalness of this notion and its relation to the usual continuous homotopy. Recently, the author and D. A. Podoprikhin have generalized Walker's notion of order isotone homotopy and suggested sufficient conditions for the preservation under such discrete homotopy (a pair of homotopies) of the property of a mapping (a pair of mappings) of ordered sets to have a fixed point (a coincidence point). This paper contains metric counterparts of the obtained results and some corollaries. The method of ordering a metric space proposed by A. Brøndsted in 1974 is used.

Keywords: fixed point, coincidence point, Brøndsted's order, order homotopy, discrete counterpart of homotopy.

REFERENCES

1. Frigon M. On continuation methods for contractive and nonexpansive mappings. *Recent Advances on Metric Fixed Point Theory*, Sevilla: Universidad de Sevilla, 1996, pp. 19–29.
2. Podoprikhin D.A., Fomenko T.N. Preservation of the existence of fixed and coincidence points under homotopy of mappings of ordered sets. *Dokl. Math.*, 2017, vol. 96, no. 3, p. 1–3. doi: 10.1134/S1064562417060199 .
3. Brøndsted A. On a lemma of Bishop and Phelps. *Pacific J. Math.*, 1974, vol. 55, no. 2, pp. 335–341.
4. Walker J.W. Isotone relations and the fixed point property for posets. *Discr. Math.*, 1984, vol. 48, no. 2–3, pp. 275–288. doi: 10.1016/0012-365X(84)90188-2 .
5. Stong R.E. Finite topological spaces. *Trans. Amer. Math. Soc.* 1966, vol. 123, no. 2, pp. 325–340. doi: 10.1090/S0002-9947-1966-0195042-2 .
6. Kirk W.A., Sims B. (eds.) *Handbook of metric fixed point theory.* NY: Springer Science & Business Media, 2001. 704 p. doi: 10.1007/978-94-017-1748-9 .
7. Podoprikhin D.A., Fomenko T.N. On coincidences of families of mappings on ordered sets. *Dokl. Math.*, 2016, vol. 94, no. 3, pp. 620–622. doi: 10.1134/S106456241606003X .
8. Fomenko T.N., Podoprikhin D. Common fixed points and coincidences of mapping families on partially ordered set. *Topology Appl.*, 2017, vol. 221, pp. 275–285. doi: 10.1016/j.topol.2016.07.024 .
9. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E. On coincidence points of mappings in partially ordered spaces. *Dokl. Math.*, 2013, vol. 88, no. 3, pp. 710–713. doi: 10.1134/S1064562413060239 .
10. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E. Coincidence points principle for mappings in partially ordered spaces. *Topology Appl.*, 2015, vol. 179, pp. 13–33. doi: 10.1016/j.topol.2014.08.013 .
11. Fomenko T.N., Podoprikhin D.A. Fixed points and coincidences of mappings of partially ordered sets. *J. Fixed Point Theory Appl.*, 2016, vol. 18, no. 4, pp. 823–842. doi: 10.1007/s11784-016-0327-7 .

12. Podoprikhin D.A. Fixed points of mappings on ordered sets. *Lobachevskii J. Math.*, 2017, vol. 38, no. 6, pp. 1069–1074.

The paper was received by the Editorial Office on June 15, 2017.

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Cite this article as:
T. N. Fomenko, Preservation of the existence of coincidence points under some discrete transformations of a pair of mappings of metric spaces, *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 4, pp. 292–300 .