Vol. 23 No. 4 **2017**

MSC: 28A80

DOI: 10.21538/0134-4889-2017-23-4-281-291

ON DENDRITES GENERATED BY POLYHEDRAL SYSTEMS AND THEIR RAMIFICATION POINTS

A. V. Tetenov, M. Samuel, D. A. Vaulin

The methods of construction of self-similar dendrites in $\mathbb{R}^d$ and their geometric properties are considered. These issues have not yet been studied in the theory of self-similar fractals. We construct and analyze a class of P-polyhedral dendrites K in $\mathbb{R}^d$, which are defined as attractors of systems $\mathcal{S} = \{S_1, \ldots, S_m\}$ of contracting similarities in $\mathbb{R}^d$ sending a given polyhedron P to polyhedra $P_i \subset P$ whose pairwise intersections either are empty or are singletons containing common vertices of the polyhedra, while the hypergraph of pairwise intersections of the polyhedra P_i is acyclic. We prove that there is a countable dense subset $G_{\mathcal{S}}(V_P) \subset K$ such that for any of its points x the local structure of a neighbourhood of x in K is defined by some disjoint family of solid angles with vertex x congruent to the angles at the vertices of P. Therefore, the ramification points of a P-polyhedral dendrite K have finite order whose upper bound depends only on the polyhedron P. We prove that the geometry and dimension of the set $CP(K)$ of the cutting points of K are defined by its main tree, which is a minimal continuum in K containing all vertices of P. That is why the dimension $\dim_H CP(K)$ of the set $CP(K)$ is less than the dimension $\dim_H(K)$ of K and $\dim_H CP(K) = \dim_H(K)$ if and only if K is a Jordan arc.

Keywords: self-similar set, dendrite, polyhedral system, main tree, ramification point, Hausdorff dimension.

REFERENCES

1. Aseev V.V., Tetenov A.V., Kravchenko A.S. On self-similar Jordan curves on the plane. *Sib. Math. J.*, 2003, vol. 44, no. 3, pp. 379–386. doi: 10.1023/A:1023848327898 .
2. Bandt C., Stahnke J. *Self-similar sets 6. Interior distance on deterministic fractals.* Preprint, Greifswald, 1990.
3. Bandt C., Keller K. Self-similar sets 2. A simple approach to the topological structure of fractals. *Math. Nachrichten*, 1991, vol. 154, pp. 27–39. doi: 10.1002/mana.19911540104 .
4. Barnsley M.F. *Fractals Everywhere* Academic Press, 1988, 396 p. ISBN: 0-12-079062-9 .
5. Charatonik J., Charatonik W. Dendrites. *Aportaciones Mat. Comun.*, 1998, vol. 22, pp. 227–253.
6. Croydon D. *Random fractal dendrites.* Ph.D. Thesis, St. Cross College, University of Oxford, Trinity, 2006. 161 p.
7. Hata M. On the structure of self-similar sets. *Japan. J. Appl. Math*, 1985, vol. 3, pp. 381–414. doi: 10.1007/BF03167083 .
8. Kigami J. Harmonic calculus on limits of networks and its application to dendrites. *J. Funct. Anal.*, 1995, vol. 128, no. 1, pp. 48–86. doi: 10.1006/jfan.1995.1023 .
9. Kigami J. *Analysis on fractals.* Cambridge: Cambridge Univer. Press, 2001, Ser. Cambridge Tracts in Math., vol. 143, 226 p. ISBN: 0-521-79321-1 .
10. Strichartz R.S. Isoperimetric estimates on Sierpinski gasket type fractals. *Trans. Amer. Math. Soc.*, 1999, vol. 351, no. 5, pp. 1705–1752. doi: 10.1090/S0002-9947-99-01999-6 .
11. Tetenov A. V. Self-similar Jordan arcs and the graph directed systems of similarities. *Siberian Math. J.*, 2006, vol. 47, no. 5, pp. 940–949. doi: 10.1007/s11202-006-0105-7 .
12. Tetenov A. V. Structural theorems in the theory of self-similar fractals : Habilitation Thesis. Gorno-Altaisk state university, Gorno-Altaisk, 2011. 216 p.

13. Zeller. R. *Branching dynamical systems and slices through fractals*, Ph.D. Thesis, University of Greifswald, 2015.

The paper was received by the Editorial Office on June 27, 2017.

Andrei Viktorovich Tetenov, Dr. Phys.-Math. Sci., Prof., Gorno-Altaisk State University, Gorno-Altaisk, 649000 Russia, e-mail: atet@mail.ru .

Mary Samuel, Department of Mathematics, Bharata Mata College, Kochi, India,
e-mail: marysamuel2000@gmail.com .

Dmitrii Alekseevich Vaulin, Gorno-Altaisk State University, Gorno-Altaisk, 649000 Russia,
e-mail: d_warrant@mail.ru .

Cite this article as:
A. V. Tetenov, M. Samuel, D. A. Vaulin, On dendrites generated by polyhedral systems and their ramification points, *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 4, pp. 281–291 .