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OPTIMAL RESULT IN A CONTROL PROBLEM WITH PIECEWISE MONOTONE DYNAMICS

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We consider an optimal control problem for a deterministic nonlinear system with piecewise monotone dynamics. The mathematical model under consideration describes the process of a chemotherapy treatment of a malignant tumor. The research makes it possible to analyze the influence of the type of nonmonotonicity on the structure of the optimal control. We consider the case when the therapy function, which describes the effect of the drug on the cell growth rate, has two maxima. Comparisons are made with the results for the previously studied case of a single maximum of the therapy function in this model. This paper is devoted to the construction of the value function for the optimal control problem under consideration. As is known, the value function is the basis for constructing an optimal synthesis, i.e., an optimal feedback strategy in the therapy. We use the fact that the value function is the unique minimax (viscosity) solution of the Cauchy problem for the basic Hamilton–Jacobi–Bellman (HJB) equation. By means of the continuous gluing of a finite number of smooth functions obtained by the Cauchy method of characteristics for auxiliary HJB equations, a continuous function φ is constructed. A new element of the construction is the line of nonsmooth gluing with the use of the Rankin–Hugoniot conditions. This line plays a key role for the optimal feedback strategy, because it determines its discontinuity line. We prove that the constructed function φ coincides with the minimax solution of the Cauchy problem for the basic HJB equation.

Keywords: optimal control, Rankine–Hugoniot line, Hamilton–Jacobi–Bellman equation, Cauchy method of characteristics.

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