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## VIRTUAL 3-MANIFOLDS OF COMPLEXITY 1 AND 2

E. A. Sbrodova, V. V. Tarkaev, E. A. Fominykh, E. V. Shumakova

Matveev in 2009 introduced the notion of virtual 3-manifold, which generalizes the classical notion of 3-manifold. A virtual manifold is an equivalence class of so-called special polyhedra. Each virtual manifold determines a 3-manifold with nonempty boundary and  $\mathbb{R}P^2$ -singularities. Many invariants of manifolds, such as Turaev–Viro invariants, can be extended to virtual manifolds. The complexity of a virtual 3-manifold is  $k$  if its equivalence class contains a special polyhedron with  $k$  true vertices and contains no special polyhedra with a smaller number of true vertices. In this paper we give a complete list of virtual 3-manifolds of complexity 1 and present two-sided bounds for the number of virtual 3-manifolds of complexity 2. The question of the complete classification for virtual 3-manifolds of complexity 2 remains open.

Keywords: virtual 3-manifold, classification, complexity.

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*Elena Aleksandrovna Sbrodova*, Cand. Sci.(Phys.-Math.), Chelyabinsk State University, Chelyabinsk, 454001 Russia, e-mail: sbrodova@csu.ru.

Vladimir Viktorovich Tarkaev, Cand. Sci.(Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia; Chelyabinsk State University, Chelyabinsk, 454001 Russia, e-mail: v.tarkaev@gmail.com.

*Evgeny Anatol'evich Fominykh*, Dr. Phys.-Math. Sci., Prof., Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia;

Chelyabinsk State University, Chelyabinsk, 454001 Russia, e-mail: efominykh@gmail.com .

*Ekaterina Valer'evna Shumakova*, Chelyabinsk State University, Chelyabinsk, 454001 Russia,  
e-mail: shumakova\_kate@mail.ru .

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