# A METANILPOTENCY CRITERION FOR A FINITE SOLVABLE GROUP

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Denote by $|x|$ the order of an element $x$ of a group. An element of a group is called primary if its order is a nonnegative integer power of a prime. If $a$ and $b$ are primary elements of coprime orders of a group, then the commutator $a^{-1}b^{-1}ab$ is called a $\star$-commutator. The intersection of all normal subgroups of a group such that the quotient groups by them are nilpotent is called the nilpotent residual of the group. It is established that the nilpotent residual of a finite group is generated by commutators of primary elements of coprime orders. It is proved that the nilpotent residual of a finite solvable group is nilpotent if and only if $|ab| \geq |a||b|$ for any $\star$-commutators of $a$ and $b$ of coprime orders.

Keywords: finite group, formation, residual, nilpotent group, commutator.

## REFERENCES

1. Huppert B. *Endliche Gruppen I.* Berlin etc.: Springer, 1967. 793 p. doi: 10.1007/978-3-642-64981-3 .

2. Bastos R., Shumyatsky P. A sufficient condition for nilpotency of the commutator subgroup. *Siberian Math. J.*, 2016, vol. 57, no. 5, pp. 762–763. doi: 10.1134/S0037446616050037 .

3. Monakhov V.S. The nilpotency criterion for the derived subgroup of a finite group [e-resource]. 2017. Preprint available at https://arxiv.org/abs/1704.01746 .

4. Schmidt O.Yu. Groups whose all subgroups are special. *Mat.Sb.*, 1924, vol. 31, pp. 366–372.

5. Monakhov V.S. *Podgruppy Shmidta, ikh sushchestvovanie i nekotorye prilozheniya* [The Schmidt subgroups, its existence, and some of their applications]. Tr. Ukrain. Mat. Congr., Kiev, 2002, Section 1, pp. 81–90.

6. Tschunichin S. Über spezielle Gruppen. *Mat. Sb.*, 1929, vol. 36, no. 2, pp. 135–137 (in Russian).

7. Doerk K., Hawkes T. *Finite soluble groups.* Berlin, N. Y.: Walter de Gruyter, 1992. 891 p. ISBN: 978-3-11-087013-8 .

8. Beidleman J., Heineken H. Minimal non-$\mathfrak{F}$-groups. *Ricerche Mat.*, 2009, vol. 58, no. 1, pp. 33-41. doi: 10.1007/s11587-009-0044-2 .

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