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ON THE OIKAWA AND ARAKAWA THEOREMS FOR GRAPHS

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The present paper is devoted to the further development of the discrete theory of Riemann surfaces, which was started in the papers by M. Baker and S. Norine at the beginning of the century. This theory considers finite graphs as analogs of compact Riemann surfaces and branched coverings of graphs as holomorphic maps. The genus of a graph is defined as the rank of its fundamental group. The main object of investigation in the paper is automorphism groups of a graph acting freely on the set of arcs. These groups are discrete analogs of groups of conformal automorphisms of a Riemann surface. The celebrated Hurwitz theorem (1893) states that the order of the group of conformal automorphisms of a compact Riemann surface of genus $g > 1$ does not exceed $84(g-1)$. Later, K. Oikawa and T. Arakawa refined this bound in the case of groups that fix several finite sets of prescribed cardinalities. This paper provides proofs of discrete versions of the mentioned theorems. In addition, a graph-theoretic version of the E. Bujalance and G. Gromadzki result improving the Arakawa theorem is obtained.

Keywords: Riemann surface, Riemann–Hurwitz formula, graph, automorphism group, harmonic map.

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