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THE STRUCTURE OF THE FIXED POINT SET OF A REDUCIBLE MONOTONE SUBHOMOGENEOUS MAPPING

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We analyze the structure of the set of nontrivial equilibria for a monotone subhomogeneous discrete-time dynamical system on the nonnegative orthant of a finite-dimensional Euclidean space under as weak additional assumptions as possible. We use the notion of local irreducibility of a nonlinear mapping introduced by the authors. It is shown that, if a monotone subhomogeneous mapping has positive fixed points lying on different rays starting at the origin, then this mapping is reducible at at least one of them and a part of the components of the mapping are positively homogeneous on segments of these rays containing the positive fixed points. In particular, for concave mappings, this means the reducibility of the mapping at zero. As a result, we obtain a generalization of the theorem on the uniqueness of the ray containing the positive fixed points of such a mapping with the only additional assumption that the mapping is irreducible on the set of its positive fixed points. In this case, the set of all positive fixed points of a monotone subhomogeneous mapping forms a continuous part of some ray starting at the origin.

Keywords: monotone mapping, subhomogeneous mapping, local irreducibility of a mapping, fixed points.

REFERENCES

1. Morishima M. *Equilibrium, Stability and Growth: A Multi-Sectoral Analysis.* Oxford University Press. 1964. 240 p. ISBN: 0-19-828145-5 . Translated to Russian under the title *Ravnovesie, ustoichivost', rost.* Moscow, Nauka Publ. 1972. 280 p.
2. Nikaido H. *Convex Structures and Economic Theory.* Mathematics in Science and Engineering. Vol. 51. New York, Academic Press. 1968. 405 p. ISBN: 1483253287 . Translated to Russian under the title *Vypuklye struktury i matematicheskaya ekonomika.* Moscow, Mir Publ. 1972. 518 p.
3. Lemmens B., Nussbaum R. D. *Nonlinear Perron—Frobenius Theory.* Cambridge Tracts in Mathematics. Vol. 189. Cambridge, Cambridge Univ. Press. 2012. 323 p. doi: 10.1017/CBO9781139026079 .
4. Krause U. *Positive dynamical systems in discrete time: theory, models and applications.* Berlin; Munich; Boston: Walter de Gruyter GmbH. 2015. 363 p. ISBN: 9783110369755 .
5. Smirnov A.I. *Ravnovesie i ustoichivost' subodnorodnykh monotonnykh diskretnykh dinamicheskikh sistem* (Equilibrium and stability of subhomogeneous monotone discrete dynamical systems). Ekaterinburg, Izd-vo Ural'skogo instituta ekonomiki upravleniya i prava. 2016. 318 p.
6. Smirnov A.I. The use of subhomogeneous monotone maps in multiplicative and additive nonlinear Perron—Frobenius theory. *Vestn. Ural. inst. ekon. upr. i prava*, 2016, no. 2(35), pp. 8–25. (in Russian)
7. Smirnov A.I. Subhomogeneous maps in the theory of monotone dynamical systems. *Vestn. Ural. inst. ekon. upr. i prava*, 2016, no. 1(34), pp. 68–80. (in Russian)
8. Smirnov A.I. Analysis of population development in conditions of non-stationary medium. In: *Methods for non-stationary problems of mathematical programming.* Sverdlovsk, IMM UNTs AN SSSR. 1979. pp. 94–103. (in Russian)
9. Takač P. Asymptotic behavior of discrete-time semigroups of sublinear, strongly increasing mappings with applications to biology. *Nonlinear Anal. Theory Meth. Appl.*, 1990, vol. 14(1), pp. 35–42.
10. Hirsch M.W., Smith H.L. Monotone Dynamical Systems. In: *Handbook of Differential Eqns: Ordinary Differential Eqns.* Canada A., Drabek P., Fonda A. (Eds.) Amsterdam, Elsevier. 2005, vol. 2, pp. 239–357. doi: 10.1016/S1874-5725(05)80006-9 .
11. Smirnov A.I. Some generalizations of the concept of irreducibility for subhomogeneous mappings. *Vestn. Ural. Inst. Ekon. Upr. i Prava*, 2016, no. 2(35), pp. 26–30 (in Russian).

12. Mazurov V.D., Smirnov A.I. Conditions for the irreducibility and primitivity of monotone subhomogeneous mappings. *Tr. Inst. Mat. i Mekh. UrO RAN*, 2016, vol. 22, no. 3, pp. 169–177 (in Russian). doi: 10.21538/0134-4889-2016-22-3-169-177 .
13. Lemmens B., Roelands M. Unique geodesics for Thompson's metric. *Ann. Institut Fourier*, 2015, vol. 65, no. 1, pp. 315–348. doi: 10.5802/aif.2932 .
14. Opoitsev V.I. *Ravnovesie i ustoichivost' v modelyakh kollektivnogo povedeniya* [Equilibrium and stability in models of collective behavior]. Moscow, Nauka Publ., 1977, 245 p.

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