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QUAZOIDS IN KNOT THEORY

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This paper is devoted to the definition and construction of quazoids, which are algebraic objects generating invariants of oriented knots and links. Such an invariant can be described in the terms of the number of proper colorings of the regions into which the diagram of a knot decomposes a 2-sphere. A coloring by elements of a set X is proper if the color diagrams of all four regions are matched by means of a function $Q\colon X \times X \times X \to X$ in the neighborhood of each double point. This function is called a quazoid over the set X. In the paper we construct two infinite series of quazoids. The first series is formed by linear quazoids over finite rings. The second series consists of quazoids generated by finite biquasiles. The invariants of knots and links generated by quazoids are nontrivial and can be used to distinguish knots. We show that all knots and links admitting diagrams with at most six double points are distinguished by linear quazoids over $\mathbb{Z}_n$, where $n \leqslant 11$.

We give results of the computer enumeration of all different quazoids over sets whose cardinality does not exceed 4.

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