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CLASSIFICATION OF LINKS OF SMALL COMPLEXITY IN A THICKENED TORUS

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The paper contains the table of links in the thickened torus $T^2 \times I$ admitting diagrams with at most four crossings. The links are constructed by a three-step process. First we enumerate all abstract regular graphs of degree 4 with at most four vertices. Then we consider all nonequivalent embeddings of these graphs into T^2. After that each vertex of each of the obtained graphs is replaced by a crossing of one of the two possible types, when a segment of the graph lies lower or above another segment. The words "above" and "lower" are understood in the sense of the coordinate of the corresponding point in the interval I. As a result, we obtain a family of diagrams of knots and links in $T^2 \times I$. We propose a number of artificial tricks that essentially reduce the enumeration and offer a rigorous proof of the completeness of the table. A generalized version of the Kauffman polynomial is used to prove that all the links are different.

Keywords: link, thickened torus, link table.

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