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ON A CHARACTERIZATION OF THE FRATTINI SUBGROUP OF A FINITE SOLVABLE GROUP

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Suppose that G is a finite solvable group, n is the length of a G -chief series of the group $F(G)/\Phi(G)$, and k is the number of central G -chief factors of this series. We prove that in this case G contains $4n - 3k$ maximal subgroups whose intersection is $\Phi(G)$. This result refines V. S. Monakhov's statement that, for any finite solvable nonnilpotent group G , its Frattini subgroup $\Phi(G)$ coincides with the intersection of all maximal subgroups M of the group G such that $MF(G) = G$. In addition, it is shown in Theorem 4.2 that the group G contains $4(n - k)$ maximal subgroups whose intersection is $\delta(G)$. The subgroup $\delta(G)$ is defined as the intersection of all abnormal maximal subgroups of G if G is not nilpotent and as G otherwise.

Keywords: finite solvable group, maximal subgroup, Frattini subgroup.

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