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## ON FINITE SIMPLE LINEAR AND UNITARY GROUPS OVER FIELDS OF DIFFERENT CHARACTERISTICS WITH COINCIDING PRIME GRAPHS. I

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Suppose that  $G$  is a finite group,  $\pi(G)$  is the set of prime divisors of its order, and  $\omega(G)$  is the set of orders of its elements. We define a graph on  $\pi(G)$  with the following adjacency relation: different vertices  $r$  and  $s$  from  $\pi(G)$  are adjacent if and only if  $rs \in \omega(G)$ . This graph is called the Gruenberg — Kegel graph or the prime graph of  $G$  and is denoted by  $GK(G)$ . In a number of articles we describe the coincidence conditions for the prime graphs of nonisomorphic simple groups. This issue is connected with Vasil'ev's Question 16.26 in the "Kourovka Notebook" about the number of nonisomorphic simple groups with the same prime graph. Earlier the author got necessary and sufficient conditions for the coincidence of the prime graphs of two nonisomorphic finite simple groups of Lie type over fields of orders  $q$  and  $q_1$ , respectively, with the same characteristic. Let  $G$  and  $G_1$  be two nonisomorphic finite simple groups of Lie type over fields of orders  $q$  and  $q_1$ , respectively, with different characteristics. The author also obtained necessary conditions for the coincidence of the prime graphs of two nonisomorphic finite simple groups of Lie type. In the present paper the latest result is refined in the case when  $G$  is a simple linear group of sufficiently high Lie rank over a field of order  $q$ . If  $G$  is a simple linear group of sufficiently high Lie rank, then we prove that the prime graphs of  $G$  and  $G_1$  may coincide only in one of nineteen cases. As corollaries of the main result, we obtain constraints (under some additional conditions) on the possible number of simple groups whose prime graph is the same as the prime graph of a simple linear group.

Keywords: finite simple linear group, finite simple unitary group, prime graph, Gruenberg — Kegel graph, spectrum.

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