DISCRETE OPERATOR RICCATI EQUATION IN AN OPTIMAL STABILIZATION PROBLEM FOR A PERIODIC LINEAR SYSTEM WITH AFTEREFFECT

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An optimal stabilization problem for linear periodic systems of differential equations with aftereffect is described in a function space. A procedure that narrows the class of admissible controls is used. Admissible feedback controls are formed in the function state space. We assume a piecewise constant periodic dependence of the controls on time. The breakpoints are independent of the choice of the states. An equivalent discrete problem of optimal stabilization in a function space is constructed. The solution of the nonautonomous discrete operator Riccati equation determines an optimal stabilizing control. The discrete stabilization problem is autonomous if the sequence of breakpoints of the controls is periodic. A representation of solutions of the autonomous discrete operator Riccati equation is found. A system of integral equations is obtained for the coefficients of this representation. A formula for the optimal stabilizing control in the discrete problem is derived.

Keywords: periodic linear system with aftereffect, optimal stabilization, discrete operator Riccati equation.

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