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AN OPTIMAL ALGORITHM FOR AN OUTERPLANAR FACILITY LOCATION PROBLEM WITH IMPROVED TIME COMPLEXITY

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We consider a network facility location problem with unbounded production levels. This problem is NP-hard in the general case and is known to have an optimal solution with quadratic complexity on a tree network. We study the case of a network representable by an outerplanar graph, i.e., by a graph whose vertices belong to one (outer) face. This problem is known to have an optimal algorithm with time complexity $O(nm^3)$, where n is the number of vertices and m is the number of possible facility locations. Using some properties of outerplanar graphs (binary 2-trees) and the existence of an optimal solution with a family of centrally connected service domains, we obtain recurrence relations for the construction of an optimal algorithm with time complexity that is smaller by a factor of \sqrt{m} than the time complexity of the earlier algorithm.

Keywords: facility location problem, network, outerplanar graph, optimal algorithm, time complexity, connectedness.

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