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AGGREGATION EQUATION WITH ANISOTROPIC DIFFUSION

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A mixed problem for the aggregation equation with anisotropic degenerating diffusion is studied. The uniqueness of the solution is proved by the method of energy estimates. For this, a special test function is constructed as a solution of an auxiliary elliptic problem. Preliminarily, we study a problem with smooth data, where the nonlocal term with convolution is replaced by a smooth vector. For this problem, we establish the nonnegativity of the solution and find an upper bound for its growth. The existence of the solution is first proved for the nondegenerate equation by a combination of the iteration method and the method of contracting mappings. Passing to the limit, we obtain a solution of the degenerate limit problem from solutions u_ε of the approximating equation. Here, we apply the compactness principle in L_1, which is similar to the principle developed in the known paper by Alt and Luckhaus. The equations under consideration appear in biological aggregation models.

Keywords: aggregation equation, anisotropic diffusion, solution existence, uniqueness of solution.

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