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EXPLICIT EXPRESSION FOR HYPERBOLIC LIMIT CYCLES OF A CLASS OF POLYNOMIAL DIFFERENTIAL SYSTEMS

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We consider systems of differential equations in the plane,

$$x' = \frac{dx}{dt} = P(x,y), \quad y' = \frac{dy}{dt} = Q(x,y),$$

where the dependent variables x and y and the independent one (the time) t are real, and $P(x,y)$, $Q(x,y)$ are polynomials in the variables x and y with real coefficients. These differential systems are mathematical models and arise in many fields of application like biology, economics, physics and engineering, etc. The existence of limit cycles is one of the more difficult objects to study in the qualitative theory of differential systems in the plane. There is a huge literature dedicated to this topic. It is known that for differential systems defined on the plane the existence of a first integral determines their phase portrait. Thus for polynomial differential systems a natural question arises: given a polynomial differential system in the plane, how to recognize if it has a first integral? There is a strong relation between the invariant algebraic curves and the theory of integrability. In this paper we introduce explicit expressions for invariant algebraic curves and for the first integral. Finally, we determine sufficient conditions for a class of polynomial differential systems to possess an explicitly given hyperbolic limit cycle. Concrete examples exhibiting the applicability of our results are introduced. The elementary method used in this paper seems to be fruitful to investigate more general planar dynamical systems in order to obtain explicitly some or all of their limit cycles at least in the case of hyperbolic cycles. In the spirit of the inverse approach to dynamical systems, we look for them as the ovals of suitably chosen invariant algebraic curves.

Keywords: planar polynomial differential system, invariant algebraic curve, first integral, limit cycle.

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