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ESTIMATES FOR BEST APPROXIMATIONS OF FUNCTIONS FROM THE LOGARITHMIC SMOOTHNESS CLASS IN THE LORENTZ SPACE

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The Lorentz space $L_{p,\tau}(\mathbb{T}^m)$ of periodic functions of m variables is considered. The Besov space $B^{0,\alpha}_{p,\tau,\theta}$ of functions with logarithmic smoothness is defined. The aim of the paper is to find the exact order of the best approximation of functions from the class $B^{0,\alpha}_{p,\tau,\theta}$ under different relations between the parameters p, τ, and θ. The paper consists of three sections. In the first section, known facts necessary for the proof of the main results are given and several auxiliary statements are proved. In the second section, order-exact estimates for the best approximation of functions from the class $B^{0,\alpha}_{p,\tau,\theta}$ are established in the space $L_{p,\tau}(\mathbb{T}^m)$. In the third section, an inequality for different metrics of trigonometric polynomials is proved and a sufficient condition for the belonging of a function $f \in L_{p,\tau_1}(\mathbb{T}^m)$ to the space $L_{p,\tau_2}(\mathbb{T}^m)$ in terms of the best approximation is established in the case $1 < \tau_2 < \tau_1$. In contrast to anisotropic Lorentz spaces, the condition is independent of the number m of the variables. Order-exact estimates for the best approximation of functions from the Besov class $B^{0,\alpha}_{p,\tau_1,\theta}$ by trigonometric polynomials $L_{p,\tau_2}(\mathbb{T}^m)$ are obtained in the case $1 < \tau_2 < \tau_1$.

Keywords: Lorentz space, Besov class, best approximation, logarithmic smoothness.

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