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UNIFORM LEBESGUE CONSTANTS OF LOCAL SPLINE APPROXIMATION

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Let a function $\varphi \in C^1[-h,h]$ be such that $\varphi(0)=\varphi'(0)=0$, $\varphi(-x)=\varphi(x)$ for $x\in[0;h]$), and $\varphi(x)$ is nondecreasing on $[0;h]$. For any function $f:\ \mathbb{R}\to\mathbb{R}$, we consider local splines of the form

$$S(x)=S_\varphi(f,x)=\sum_{j\in\mathbb{Z}} y_j B_\varphi\Big(x+\frac{3h}{2}-jh\Big)\quad (x\in\mathbb{R}),$$

where $y_j=f(jh)$, $m(h)>0$, and

$$B_\varphi(x)=m(h)\begin{cases}\varphi(x), & x\in[0;h],\\ 2\varphi(h)-\varphi(x-h)-\varphi(2h-x), & x\in[h;2h],\\ \varphi(3h-x), & x\in[2h;3h],\\ 0, & x\notin[0;3h].\end{cases}$$

These splines become parabolic, exponential, trigonometric, etc., under the corresponding choice of the function φ. We study the uniform Lebesgue constants $L_\varphi=\|S\|_C^C$ (the norms of linear operators from C to C) of these splines as functions depending on φ and h. In some cases, the constants are calculated exactly on the axis $\mathbb{R}$ and on a closed interval of the real line (under a certain choice of boundary conditions from the spline $S_\varphi(f,x)$).

Keywords: Lebesgue constants, local splines, three-point system.

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