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SOLVABILITY OF THE GENERALIZED TRAVELING SALESMAN PROBLEM IN THE CLASS OF QUASI- AND PSEUDOPYRAMIDAL TOURS

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We consider the general setting of the Generalized Traveling Salesman Problem (GTSP), where, for a given weighted graph and a partition of its nodes into clusters (or megalopolises), it is required to find a cheapest cyclic tour visiting each cluster exactly once. Generalizing the classical notion of pyramidal tour, we introduce quasi- and pseudopyramidal tours for the GTSP and show that, for an arbitrary instance of the problem with n nodes and k clusters, optimal l -quasi-pyramidal and l -pseudopyramidal tours can be found in time $O(4^l n^3)$ and $O(2^l k^{l+4} n^3)$, respectively. As a consequence, we prove that the GTSP belongs to the class FTP with respect to parametrizations given by such types of routes. Furthermore, we establish the polynomial-time solvability of the geometric subclass of the problem known in the literature as GTSP-GC, where an arbitrary statement is subject to the additional constraint $H \leq 2$ on the height of the grid defining the clusters.

Keywords: Generalized Traveling Salesman Problem (GTSP), polynomially solvable subclass, quasi-pyramidal tour, pseudopyramidal tour.

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