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APPROXIMATION OF THE MEASURE OF A CONVEX COMPACT SET

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We consider an approach to constructing upper and lower bounds for the measure of a convex compact set. The approach is based on extremal inscribed and circumscribed parallelepipeds. It is assumed that the measure of a parallelepiped can be easily calculated. It is shown that the problem of constructing an inscribed parallelepiped of maximum volume is reduced to a convex programming problem with exponential number of constraints. In some particular important cases the exponential number of constraints can be avoided. We suggest an algorithm for the iterative inner and outer approximation of a convex compact set by parallelepipeds. The complexity of the algorithm is estimated. The results of a preliminary numerical experiment are given. The possibility of constructing parallelepipeds that are extremal with respect to measure is discussed. Some advantages of the proposed approach are specified in the conclusion.

Keywords: measure, convex compact set, extremal parallelepiped, inner and outer approximation.

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