UNIFORM APPROXIMATION OF THE CURVATURE OF SMOOTH PLANAR CURVES WITH THE USE OF PARTIAL SUMS OF FOURIER SERIES

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An error bound for the approximation of the curvature of graphs of periodic functions from the class W^r for $r \geq 3$ in the uniform metric is obtained with the use of the simplest approximation technique for smooth periodic functions, which is approximation by partial sums of their trigonometric Fourier series. From the mathematical point of view, the interest in this problem is connected with the specific nonlinearity of the graph curvature operator on the class of smooth functions W^r on a period or a closed interval for $r \geq 2$. There are several papers on curvature approximation for planar curves in the mean-square and Chebyshev norms. In previous works, the approximation was performed by partial sums of trigonometric series (in the L^2 norm), interpolation splines with uniform knots, Fejér means of partial sums of trigonometric series, and orthogonal interpolating wavelets based on Meyer wavelets (in the C^∞ norm). The technique of this paper, based on the lemma, can possibly be generalized to the L^p metric and other approximation methods.

Keywords: curvature approximation, planar curves from the class W^r, uniform metric.

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