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SPARSE TRIGONOMETRIC APPROXIMATION OF BESOV CLASSES OF FUNCTIONS WITH SMALL MIXED SMOOTHNESS

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We consider problems concerned with finding order-exact estimates for a sparse trigonometric approximation, more exactly, for the best m-term trigonometric approximation $\sigma_m(F)_q$, where F are the Nikol'skii–Besov classes $\mathbf{MB}^r_{p,\theta}$ of functions with mixed smoothness and classes of functions close to them. Attention is paid to relations between the parameters p and q for $1 < p < q < \infty$ and $q > 2$. In 2003 Romanyuk found order-exact estimates of $\sigma_m(\mathbf{MB}^r_{p,\theta})_q$ for $1 \le \theta \le \infty$ (the upper estimates are nonconstructive) in the cases $1 < p \le 2 < q < \infty$, $r > 1/p - 1/q$ and $2 < p < q < \infty$, $r > 1/2$. Complementing Romanyuk's studies, Temlyakov has recently found constructive upper estimates (provided by a constructive method based on a greedy algorithm) for $\sigma_m(\mathbf{MB}^r_{p,\theta})_q \asymp \sigma_m(\mathbf{MH}^r_{p,\theta})_q$, $1 \le \theta \le \infty$, in the case of great smoothness, i.e., for $1 < p < q < \infty$, $q > 2$, and $r > \max\{1/p; 1/2\}$; he considered wider classes $\mathbf{MH}^r_{p,\theta}$ ($\mathbf{MB}^r_{p,\theta} \subset \mathbf{MH}^r_{p,\theta} \subset \mathbf{MH}^r_p$, $1 \le \theta < \infty$). Less attention was paid to constructive upper estimates of the values $\sigma_m(\mathbf{MB}^r_{p,\theta})_q$ and $\sigma_m(\mathbf{MH}^r_{p,\theta})_q$ in the case of small smothness, i.e., for $1 < p \le 2 < q < \infty$ and $1/p - 1/q < r \le 1/p$. For $1 < p \le 2 < q < \infty$ Temlyakov found a constructive upper estimate for $\sigma_m(\mathbf{MB}^r_{p,\theta})_q$ in the cases $\theta = \infty$, $1/p - 1/q < r < 1/p$ and $\theta = p$, $(1/p - 1/q)q' < r < 1/p$, where $1/q + 1/q' = 1$, while the author found a constructive upper estimate for $\sigma_m(\mathbf{MH}^r_{p,\theta})_q$ if $r = 1/p$ and $p \le \theta \le \infty$; it turned out that $\sigma_m(\mathbf{MH}^r_{p,\theta})_q \asymp \sigma_m(\mathbf{MB}^r_{p,\theta})_q(\log m)^{1/\theta}$ for $r = 1/p$ and $p \le \theta < \infty$. In the present paper, we derive a constructive upper estimate for $\sigma_m(\mathbf{MB}^r_{p,\theta})_q$ (or $\sigma_m(\mathbf{MH}^r_{p,\theta})_q$) for $1 < p \le 2 < q < \infty$ and $(1/p - 1/q)q' < r < 1/p$ when $p < \theta < \infty$ (or $p \le \theta < \infty$) as well as order-exact (though nonconstructive upper) estimates for the values $\sigma_m(\mathbf{MB}^r_{p,\theta})_q$, $2 < p < q < \infty$, $\theta = 1$, $r = 1/2$, and $\sigma_m(\mathbf{MH}^r_{p,\theta})_q$, $1 < p \le 2 < q < \infty$, $1 \le \theta < p$, $r = 1/p$, which complement Romanyuk's results and the author's recent results, respectively.

Keywords: nonlinear approximation, sparse trigonometric approximation, mixed smoothness, Besov classes, exact order bounds.

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