TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 23 No. 3 **2017**

MSC: 97N50

DOI: 10.21538/0134-4889-2017-23-3-224-233

CONVERGENCE BOUNDS FOR SPLINES FOR THREE-POINT RATIONAL INTERPOLANTS OF CONTINUOUS AND CONTINUOUSLY DIFFERENTIABLE FUNCTIONs

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For functions $f(x)$ continuous on an interval $[a,b]$ and grids of pairwise different nodes $\Delta\colon a = x_0 < x_1 < \cdots < x_N = b$ $(N \geqslant 2)$, we study the convergence rate of piecewise rational functions $R_{N,1}(x) = R_{N,1}(x,f)$ such that, for $x \in [x_{i-1}, x_i]$ $(i = 1, 2, \ldots, N)$, we have $R_{N,1}(x) = (R_i(x)(x - x_{i-1}) + R_{i-1}(x)(x_i - x))/(x_i - x_{i-1})$, where $R_i(x) = \alpha_i + \beta_i(x - x_i) + \gamma_i/(x - g_i)$ $(i = 1, 2, \ldots, N-1)$; the coefficients α_i, β_i, and γ_i are defined by the conditions $R_i(x_j) = f(x_j)$ for $j = i-1, i, i+1$; and the poles g_i are defined by the nodes. It is assumed that $R_0(x) \equiv R_1(x)$ and $R_N(x) \equiv R_{N-1}(x)$. Bounds for the convergence rate of $R_{N,1}(x,f)$ are found in terms of certain structural characteristics of the function:

(1) the third-order modulus of continuity in the case of uniform grids;

(2) the variation and the modulus of change of the first and second derivatives in the case of continuously differentiable functions $f(x)$; here, the bounds in terms of the variation have the order of the best polynomial spline approximations.

Keywords: splines, interpolation splines, rational splines.

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The paper was received by the Editorial Office on April 17, 2017.

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Cite this article as:
A.-R. K. Ramazanov, V. G. Magomedova, Convergence bounds for splines for three-point rational interpolants of continuous and continuously differentiable functions, *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 3, pp. 224–233 .