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THREE EXTREMAL PROBLEMS IN THE HARDY AND BERGMAN SPACES OF FUNCTIONS ANALYTIC IN A DISK

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Let a nonnegative measurable function $\gamma(\rho)$ be nonzero almost everywhere on $(0, 1)$, and let the product $\rho\gamma(\rho)$ be summable on $(0, 1)$. Denote by $\mathcal{B} = B_{\gamma}^{p,q}$, $1 \leq p \leq \infty$, $1 \leq q < \infty$, the space of functions f analytic in the unit disk for which the function $M_p^q(f, \rho)\rho\gamma(\rho)$ is summable on $(0, 1)$, where $M_p^q(f, \rho)$ is the p -mean of f on the circle of radius ρ ; this space is equipped with the norm

$$\|f\|_{B_{\gamma}^{p,q}} = \|M_p(f, \cdot)\|_{L_{\rho\gamma(\rho)}^q(0,1)}.$$

In the case $q = \infty$, the space $\mathcal{B} = B_{\gamma}^{p,\infty}$ is identified with the Hardy space H^p . Using an operator L given by the equality $Lf(z) = \sum_{k=0}^{\infty} l_k c_k z^k$ on functions $f(z) = \sum_{k=0}^{\infty} c_k z^k$ analytic in the unit disk, we define the class

$$LB_{\gamma}^{p,q}(N) := \{f : \|Lf\|_{B_{\gamma}^{p,q}} \leq N\}, \quad N > 0.$$

For a pair of such operators L and G , under some constraints, the following three extremal problems are solved.

(1) The best approximation of the class $LB_{\gamma}^{p_1,q_1}(1)$ by the class $GB_{\gamma}^{p_3,q_3}(N)$ in the norm of the space $B_{\gamma}^{p_2,q_2}$ is found for $2 \leq p_1 \leq \infty$, $1 \leq p_2 \leq 2$, $1 \leq p_3 \leq 2$, $1 \leq q_1 = q_2 = q_3 \leq \infty$, and $q_s = 2$ or ∞ .

(2) The best approximation of the operator L by the set $\mathcal{L}(N)$, $N > 0$, of linear bounded operators from $B_{\gamma}^{p_1,q_1}$ to $B_{\gamma}^{p_2,q_2}$ with the norm not exceeding N on the class $GB_{\gamma}^{p_3,q_3}(1)$ is found for $2 \leq p_1 \leq \infty$, $1 \leq p_2 \leq 2$, $2 \leq p_3 \leq \infty$, $1 \leq q_1 = q_2 = q_3 \leq \infty$, and $q_s = 2$ or ∞ .

(3) Bounds for the modulus of continuity of the operator L on the class $GB_{\gamma}^{p_3,q_3}(1)$ are obtained, and the exact value of the modulus is found in the Hilbert case.

Keywords: Hardy and Bergman spaces, best approximation of a class by a class, best approximation of an unbounded operator by bounded operators, modulus of continuity of an operator.

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