REGULARIZATION METHODS AND ISSUES OF LEXICOGRAPHIC CORRECTION FOR CONVEX PROGRAMMING PROBLEMS WITH INCONSISTENT CONSTRAINTS

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We consider convex programming problems for which it is unknown in advance whether their constraints are consistent. For the numerical analysis of these problems, we propose to apply a multistep symmetric regularization of the classical Lagrange function with respect to both primal and dual variables and then to solve the arising minimax problems with a small parameter. The latter problems are always solvable and give either normal decisions of the original problems in the case of their propriety or, in the improper case, generalized solutions that minimize the discrepancies of the constraints and optimize the value of the objective function asymptotically with respect to the parameter. Minimax problems can also form a basis for the construction of new duality diagrams in convex programming, at least for improper settings. Regularization diagrams are provided, a primal minimax setting is written, theorems on the convergence and numerical stability of the method are proved, and an informal interpretation of the generalized solutions is given. The study develops the authors' earlier results obtained for linear programming problems.

Keywords: convex programming, duality, generalized solutions, regularization method, penalty function method.

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