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UNIFORM APPROXIMATION BY PERFECT SPLINES

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The problem of uniform approximation of a continuous function on a closed interval is considered. In the case of approximation by the class $W^{(n)}$ of functions whose nth derivative is bounded by 1 almost everywhere, a criterion for a best approximation element is known. This criterion, in particular, requires that the approximating function coincide on some subinterval with a perfect spline of degree n with finitely many knots. Since perfect splines belong to the class $W^{(n)}$, we study the following restriction of the problem: a continuous function is approximated by the set of perfect splines with an arbitrary finite number of knots. We establish the existence of a perfect spline that is a best approximation element both in $W^{(n)}$ and in this set. This means that the values of best approximation in the problems are equal. We also show that the best approximation elements in this set satisfy a criterion similar to the criterion of best approximation in $W^{(n)}$. The set of perfect splines is shown to be everywhere dense in $W^{(n)}$.

Keywords: uniform approximation, functions with bounded derivative, perfect splines.

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