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# THE CONNECTION BETWEEN INFINITE-DIMENSIONAL STOCHASTIC PROBLEMS AND PROBLEMS FOR PROBABILISTIC CHARACTERISTICS

**I. V. Mel'nikova, U. A. Alekseeva, V. A. Bovkun**

We study the connection between the Cauchy problem for infinite-dimensional quasi-linear stochastic equations with multiplicative Wiener process and the (direct and inverse) Cauchy problems for the corresponding deterministic partial differential equations (with Fréchet derivatives). For Markov processes given by stochastic equations, we prove the existence of two limits defined in terms of densities of transition probabilities; these limits generalize to the general case the average values and covariances of these processes. A partial differential equation, which is an infinite-dimensional analog of the Kolmogorov equation, is obtained for probabilistic characteristics of the processes with coefficients defined by these limits. The fact that the solutions of the stochastic differential equations are infinite-dimensional has a profound effect on the expressions for the limits and for the obtained partial differential equations. The form of these expressions is different as compared to the finite-dimensional case: the equations contain a smooth potential, which, in a sense, plays the role of test functions in the equations considered as generalized ones.

Keywords: stochastic Cauchy problem, Q-Wiener process, Markov process, semigroup generator, Kolmogorov equation.

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*Irina Valer'yanovna Melnikova,* Dr. Phys.-Math. Sci., Prof., Ural Federal University, Institute of Natural Sciences and Mathematics, Yekaterinburg, 620002 Russia, e-mail: Irina.Melnikova@urfu.ru .

*Ul'yana Alekseevna Alekseeva,* Cand. Sci. (Phys.-Math.), Ural Federal University, Institute of Natural Sciences and Mathematics, Yekaterinburg, 620002 Russia, e-mail: Uliana.Alekseeva@urfu.ru .

*Vadim Andreevich Bovkun,* doctoral student, Ural Federal University, Institute of Natural Sciences and Mathematics, Yekaterinburg, 620002 Russia, e-mail: 123456m@inbox.ru .

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