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COMPUTATIONAL COMPLEXITY FOR THE PROBLEM OF OPTIMAL INTERSECTIONS OF STRAIGHT LINE SEGMENTS BY DISKS

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Computational complexity and exact polynomial algorithms are reported for the problem of stabbing a set of straight line segments with a least cardinality set of disks of fixed radii $r > 0$, where the set of segments forms a straight line drawing $G = (V, E)$ of a planar graph without edge crossings. Similar geometric problems arise in network security applications (Agarwal et al., 2013). We establish the strong NP-hardness of the problem for edge sets of Delaunay triangulations, Gabriel graphs, and other subgraphs (which are often used in network design) for $r \in [d_{\min}, \eta d_{\max}]$ and some constant η, where $d_{\max}$ and $d_{\min}$ are the Euclidean lengths of the longest and shortest graph edges, respectively.

Keywords: computational complexity, Hitting Set Problem, Continuous Disk Cover problem, Delaunay triangulations.

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