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POWER WIGHT INTEGRABILITY FOR SUMS OF MODULI OF BLOCKS FROM TRIGONOMETRIC SERIES

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The following problem is studied: find conditions on sequences $\{\gamma(r)\}$, $\{n_j\}$, and $\{v_j\}$ under which, for any sequence $\{b_k\}$ such that $\sum_{k=r}^{\infty}|b_k - b_{k+1}| \le \gamma(r)$, $b_k \to 0$, the integral $\int_0^{\pi} U^p(x)/x^q dx$ is convergent, where $p > 0$, $q \in [1-p; 1)$, and $U(x) := \sum_{j=1}^{\infty}\left|\sum_{k=n_j}^{v_j} b_k \sin kx\right|$. In the case $\gamma(r) = B/r$, $B > 0$, this problem was studied and solved by S. A. Telyakovskii. In the case where $p \ge 1$, $q = 0$, $v_j = n_{j+1} - 1$, and the sequence $\{b_k\}$ is monotone, A. S. Belov obtained a criterion for the belonging of the function $U(x)$ to the space L_p. In Theorem 1 of the present paper, we give sufficient conditions for the convergence of the above integral, which for $\gamma(r) = B/r$, $B > 0$, coincide with Telyakovskii's sufficient conditions. In the case $\gamma(r) = O(1/r)$, Telyakovskii's conditions may be violated, but the application of Theorem 1 guarantees the convergence of the integral. The corresponding examples are given in the last section of the paper. The question on necessary conditions for the convergence of the integral $\int_0^{\pi} U^p(x)/x^q dx$, where $p > 0$ and $q \in [1-p; 1)$, remains open.

Keywords: trigonometric series, sums of moduli of blocks, power weight.

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