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CONTACT RESISTANCE OF A SQUARE CONTACT

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We consider a conductive body in the form of a parallelepiped with small square contacts attached to its ends. The potential of the electric current is modelled by a boundary value problem for the Laplace equation in a parallelepiped. The zero normal derivative is assigned on the boundary except for the areas under the contacts, where the derivative is a nonzero constant. Physically, this condition corresponds to the presence of a low-conductivity film on the surface of the contacts. The problem is solved by separation of variables, and then the electrical resistance is found as a functional of the solution in the form of the sum of a double series. Our main aim is to study the dependence of the resistance on a small parameter characterizing the size of the contacts. The leading term of the asymptotics that expresses this dependence is the contact resistance. The mathematical problem is to treat the singular dependence of the sum of the series corresponding to the resistance on the small parameter: the series diverges as the small parameter vanishes. We solve this problem by replacing the series with a two-dimensional integral. We find the leading term of the asymptotics and estimate the remainder. It turns out that the main contribution to the remainder is made by the difference between the two-dimensional integral and the double sum.

Keywords: contact resistance, boundary value problem, electric potential, Laplace equation, small parameter.

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