TWO-PARAMETER ASYMPTOTICS IN A BISINGULAR CAUCHY PROBLEM FOR A PARABOLIC EQUATION

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The Cauchy problem for a quasilinear parabolic equation with a small parameter ε at the highest derivative is considered. The initial function, which has the form of a smoothed step, depends on a "stretched" variable x/ρ, where ρ is another small parameter. This problem statement is of interest in applications as a model of propagation of nonlinear waves in physical systems in the presence of small dissipation. In the case corresponding to a compression wave, asymptotic solutions of the problem are constructed in the parameters ε and ρ independently tending to zero. It is assumed that $\varepsilon/\rho \to 0$. Far from the line of discontinuity of the limit solution, asymptotic solutions are constructed in the form of series in powers of ε and ρ. In a small domain of linear approximation, an asymptotic solution is constructed in the form of a series in powers of the ratio ρ/ε. The coefficients of the inner expansion are found from a recurrence chain of initial value problems. The asymptotics of these coefficients at infinity is studied. The time of reconstruction of the scale of the inner space variable is found.

Keywords: parabolic equation, Cauchy problem, asymptotics.

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