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ASYMPTOTICS OF THE VELOCITY POTENTIAL OF AN IDEAL FLUID FLOWING AROUND A THIN BODY

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We consider the Neumann problem outside a small neighborhood of a planar disk in three-dimensional space. The surface of this neighborhood is assumed to be smooth, and its thickness is characterized by a small parameter ε. A uniform asymptotic expansion of the solution of this problem with respect to ε is constructed by the matching method. Since the problem turned out to be bisingular, an additional inner asymptotic expansion in the so-called stretched variables was constructed near the edge of the disk. A physical interpretation of the solution of this boundary value problem is the velocity potential of a laminar flow of an ideal fluid around a thin body, which is the neighborhood of the disk. It is assumed that this flow has unit velocity at a large distance from the disk, which is equivalent to the following condition for the potential: $u(x_1, x_2, x_3, \varepsilon) = x_3 + O(r^{-2})$ as $r \to \infty$, where r is the distance to the origin. The boundary condition of this problem is the impermeability of the surface of the body: $\partial u/\partial \mathbf{n} = 0$ at the boundary. After subtracting x_3 from the solution $u(x_1, x_2, x_3, \varepsilon)$, we get a boundary value problem for the potential $\widetilde{u}(x_1, x_2, x_3, \varepsilon)$ of the perturbed flow of the motion. Since the integral of the function $\partial\widetilde{u}/\partial\mathbf{n}$ over the surface of the body is zero, we have $\widetilde{u}(x_1, x_2, x_3, \varepsilon) = O(r^{-2})$ as $r \to \infty$. Hence, all the coefficients of the outer asymptotic expansion with respect to ε have the same behavior at infinity. However, these coefficients have increasing singularities at the approach to the edge of the disk, which implies the bisingularity of the problem.

Keywords: boundary value problem, Laplace equation, asymptotic expansion, matching method, thin body, laminar flow, ideal fluid.

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