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ASYMPTOTICS OF A SOLUTION TO A SINGULARLY PERTURBED TIME-OPTIMAL CONTROL PROBLEM

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In the study of singularly perturbed optimal control problems, asymptotic solutions to the boundary value problem resulting from the optimality condition for the control are constructed by means of the well-known and well-developed method of boundary functions. This approach is effective for problems with smooth controls from an open domain. Problems with a closed bounded domain of the control have been investigated less thoroughly. The cases that are usually considered involve situations where the control is a scalar function or a multidimensional function with values in a convex polyhedron. In the latter case, since the optimal control is a piecewise constant function with values at the vertices of the polyhedron, it is important to describe the asymptotic behavior of the switching points of the optimal control. In this paper we investigate a time-optimal control problem for a singularly perturbed linear autonomous system with smooth geometric constraints on the control in the form of a ball. The main difference of this case from systems with fast and slow variables studied earlier is that in this case the matrix at the fast variables is a multidimensional analog of the second-order Jordan cell with zero eigenvalue and, thus, does not satisfy the standard condition of asymptotic stability. The solvability of the problem is proved. Power asymptotic expansions of the optimal time and optimal control with respect to a small parameter at the derivatives in the equations of the system are constructed and substantiated.

Keywords: optimal control, time-optimal control problem, asymptotic expansion, singularly perturbed problems, small parameter.

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