

DOI: 10.21538/0134-4889-2017-23-2-42-66

MSC: 93C70, 49N05, 34E05, 34E10, 34K26, 34K28, 35K15, 35K59, 35C20

THE YEKATERINBURG HERITAGE OF ARLEN MIKHAILOVICH IL'IN

A. R. Danilin, S. V. Zakharov, O. O. Kovrizhnykh,
E. F. Lelikova, I. V. Pershin, O. Yu. Khachai

The main problems formulated by A. M. Il'in and solved by his disciples working now in Yekaterinburg are considered. These problems are related to the method of matched asymptotic expansions used for finding asymptotic solutions of equations with a singular dependence on a small parameter. In addition to boundary value problems for equations of mathematical physics, we consider systems of nonlinear equations and systems of linear equations depending on two small parameters. We also consider problems of finding asymptotic expansions for fundamental solutions of parabolic equations and optimal control problems depending on a small parameter.

Keywords: singularly perturbed problems, asymptotic expansions, small parameter, method of matched asymptotic expansions, optimal control.

REFERENCES

1. Il'in A.M. *Matching of asymptotic expansions of solutions of boundary value problems*. Providence, American Mathematical Society, 1992, 281 p. ISBN: 978-0-8218-4561-5. Original Russian text published *Soglasovanie asimptoticheskikh razlozhenij reshenij kraevykh zadach*, Moscow, Nauka Publ., 1989, 336 p.
2. Il'in A.M., Lelikova E.F. A method of joining asymptotic expansions for the equation $\varepsilon\Delta u - a(x, y)u_y = f(x, y)$ in a rectangle. *Math. USSR-Sb.*, 1975, vol. 25, no. 4, pp. 533–548. doi: 10.1070/SM1975v025n04ABEH002461 .
3. Lelikova E.F. On the asymptotics of a solution of a second order elliptic equation with small parameter at a higher derivative. *Proc. Steklov Inst. Math.*, 2003, suppl. 1, pp. S129–S143.
4. Lelikova E.F. The asymptotics of the solution of an equation with a small parameter in a domain with angular points. *Sbornik Math.*, 2010, vol. 201, no. 10, pp. 1495–1510. doi: 10.1070/SM2010v201n10ABEH004119 .
5. Lelikova E.F. On the asymptotics of a solution of a second order elliptic equation with a small parameter multiplying one of the highest order derivatives. *Trans. Moscow Math. Soc.*, vol. 71, 2010, pp. 141–174. ISSN: 1547-738X.
6. Lelikova E.F. On the asymptotic behavior of a solution to an equation with a small parameter in a neighborhood of a boundary inflection point. *Dokl. Math.*, 2012, vol. 86, no. 3, pp. 756–759.
7. Pershin I.V. Construction of the asymptotics of the Green function in a neighborhood of a singular point. *Differ. Equ.*, 2001, vol. 37, no. 6, pp. 883–885. doi: 10.1023/A:1019211224598 .
8. Pershin I.V. Asymptotics of a solution to the heat equation with a singularity at the boundary. *Trudy Inst. Mat. Mekh. UrO RAN*, 2012, vol. 18, no. 1, pp. 268–272 (in Russian).
9. Zakharov S.V. The Cauchy problem for a quasilinear parabolic equation with a large initial gradient and low viscosity. *Comput. Math. Math. Phys.*, 2010, vol. 50, no. 4, pp. 665–672. doi: 10.1134/S0965542510040081 .
10. Il'in A.M., Zakharov S.V. From weak discontinuity to gradient catastrophe. *Sb. Math.*, 2001, vol. 192, no. 10, pp. 1417–1433. doi: 10.1070/SM2001v192n10ABEH000599 .

11. Zakharov S.V. Asymptotic solution of a Cauchy problem in a neighbourhood of a gradient catastrophe. *Sbornik Math.*, 2006, vol. 197, no. 6, pp. 835–851. doi: 10.1070/SM2006v197n06ABEH003780.
12. Zakharov S.V. Singularities of A and B types in asymptotic analysis of solutions of a parabolic equation. *Funct. Anal. Appl.*, 2015, vol. 49, no. 4, pp. 307–310. doi: 10.1007/s10688-015-0120-1.
13. Il'in A.M., Khachay O.Y. Singular initial value problem for a system of ordinary differential equations with a small parameter. *Dokl. Math.*, 2008, vol. 78, no. 2, pp. 729–732. doi: 10.1134/S1064562408050232.
14. Il'in A.M., Leonychev Y.A., Khachay O.Y. The asymptotic behaviour of the solution to a system of differential equations with a small parameter and singular initial point. *Sbornik Math.*, 2010, vol. 201, no. 1, pp. 79–101. doi: 10.1070/SM2010v201n01ABEH004066.
15. Il'in A.M., Khachay O.Y. Structure of boundary layers in singular problems. *Dokl. Math.*, 2012, vol. 86, no. 1, pp. 497–499. doi: 10.1134/S1064562412040187.
16. Il'in A.M., Kovrizhnykh O.O. The asymptotic behavior of solutions to systems of linear equations with two small parameters. *Dokl. Math.*, 2004, vol. 69, no. 3, pp. 336–337.
17. Kovrizhnykh O.O. Asymptotic expansion of a solution of a singularly perturbed system of linear equations. *Differ. Equ.*, 2005, vol. 41, no. 10, pp. 1392–1402. doi: 10.1007/s10625-005-0291-2.
18. Il'in A.M., Danilin A.R. *Asimptoticheskie metody v analize* [Asymptotic methods in analysis]. Moscow, Fizmatlit Publ., 2009, 248 p. ISBN: 978-5-9221-1056-3/hbk.
19. Danilin A.R. Asymptotic behavior of the optimal cost functional for a rapidly stabilizing indirect control in the singular case. *Comput. Math. Math. Phys.*, 2006, vol. 46, no. 12, pp. 2068–2079. doi: 10.1134/S0965542506120062.
20. Danilin A.R., Kovrizhnykh O.O. Time-optimal control of a small mass point without environmental resistance. *Dokl. Math.*, 2013, vol. 88, no. 1, pp. 465–467. doi: 10.1134/S1064562413040364.
21. Danilin A.R., Zorin A.P. Asymptotic expansion of solutions to optimal boundary control problems. *Dokl. Math.*, 2011, vol. 84, no. 2, pp. 665–668. doi: 10.1134/S106456241106024X.
22. Danilin A.R. Optimal boundary control in a small concave domain. *Ufimsk. Mat. Zh.*, 2012, vol. 4, no. 2, pp. 87–100 (in Russian).

The paper was received by the Editorial Office on February 1, 2017.

Aleksei Rufimovich Danilin, Dr. Phys.-Math. Sci., Prof., Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia; Ural Federal University, Yekaterinburg, 620002 Russia, e-mail: dar@imm.uran.ru.

Sergei Viktorovich Zakharov, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia, e-mail: svz@imm.uran.ru.

Ol'ga Olegovna Kovrizhnykh, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia; Ural Federal University, Yekaterinburg, 620002 Russia, e-mail: koo@imm.uran.ru.

Elena Fedorovna Lelikova, Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia; Ural Federal University, Yekaterinburg, 620002 Russia, e-mail: lef@imm.uran.ru.

Igor' Viktorovich Pershin, Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia, e-mail: piv@imm.uran.ru.

Oleg Yur'evich Khachay, Cand. Sci. (Phys.-Math.), Ural Federal University, Yekaterinburg, 620002 Russia, e-mail: khachay@yandex.ru.

Cite this article as:

A. R. Danilin, S. V. Zakharov, O. O. Kovrizhnykh, E. F. Lelikova, I. V. Pershin, O. Yu. Khachay, the Yekaterinburg heritage of Arlen Mikhailovich Il'in, *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 2, pp. 42–66.