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ANALYTIC SOLUTIONS OF STATIONARY COMPLEX CONVECTION DESCRIBING A SHEAR STRESS FIELD OF DIFFERENT SIGNS

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We study layered convection of a viscous incompressible fluid. The flow of an incompressible medium is described by the overdetermined system of the Oberbeck–Boussinesq equations. An exact solution of the overdetermined system of equations is found. The solution belongs to the Lin–Sidorov–Aristov class. In this class the velocities are homogeneous with respect to the horizontal variables. The pressure and temperature fields are linear functions of the coordinates x and y. The use of the Lin–Sidorov–Aristov class preserves the nonlinearity of the motion equations only in the heat equation. The boundary value problem is studied for the Bénard–Marangoni convection with heat transfer at the free boundary. The heat transfer is determined by the Newton–Richman law. The convective motion of a fluid is characterized by the existence of a layer thickness at which the friction force (the shear stress) vanishes at an interior point of the fluid layer. We give constraints on the control parameters that determine the no-slip conditions for the layers in the cases of thermal and solutal convective flows.

Keywords: Bénard–Marangoni convection, exact solution, boundary condition of the third kind, shear stress.

REFERENCES

1. Lin C.C. Note on a class of exact solutions in magneto-hydrodynamics. *Arch. Rational Mech. Anal.*, 1958, vol. 1, pp. 391–395.
2. Sidorov A. F. On a class of solutions of the equations of gas dynamics and natural convection. *Numerical and analytical methods for solving problems of continuum mechanics* / ed. A.F. Sidorov, Yu.N. Kondyurin, Sverdlovsk, Ural. Nauchn. Tsentr Akad. Nauk SSSR, 1981, pp. 101–117.
3. Korotkii A.I. et [al.]. On the development of analytical and numerical solution methods for problems of continuum mechanics. *Trudy Inst. Mat. Mekh. UrO RAN*, 2013, vol. 19, no. 2, pp. 203–215.
4. Aristov S. N., Prosviryakov E.Yu. A new class of exact solutions for three-dimensional thermal diffusion equations. *Theor. Found. Chem. Eng.*, 2016, vol. 50, no. 3, pp. 286–293. doi: 10.1134/S0040579516030027 .
5. Pukhnachev V.V. Symmetries in the Navier–Stokes equations. *Usp. Mekh.*, 2006, vol. 3, no. 6, pp. 3–76.
6. Ryzhkov I.I. *Termodiffuziya v smesyakh: uravneniya, simmetrii, resheniya i ikh ustojchivost'* [Thermal diffusion in mixtures: equations, symmetries, solutions and their stability]. Novosibirsk, Publishing house of SB RAS, 2013, 200 p.
7. Sidorov A.F. Two classes of solutions of the fluid and gas mechanics equations and their connection to traveling wave theory. *J. Appl. Mech. Tech. Phy.*, 1989, vol. 30, no. 2, pp. 197–203. doi: 10.1007/BF00852164 .
8. Aristov S.N., Prosviryakov E.Y., Spevak L.F. Nonstationary laminar thermal and solutal Marangoni convection of a viscous fluid. *Computational Continuum Mechanics*, 2015, vol. 8, no. 4, pp. 445–456. doi: 10.7242/1999-6691/2015.8.4.38 (in Russian).

9. Aristov S. N., Prosviryakov E.Yu., Spevak L.F. Unsteady-state Bénard-Marangoni convection in layered viscous incompressible flows. *Theor. Found. Chem. Eng.*, 2016, vol. 50, no. 2, pp. 132–141. doi: 10.1134/S0040579516020019 .
10. Aristov S.N., Shvarts K.G. *Vikhrevye techeniya v tonkikh sloyakh zhidkosti* [Vortex flow in thin liquid layers]. Kirov: Vyatka State Univ. Publ., 2011, 207 p.
11. Landau L.D., Lifshitz E.M. Fluid Mechanics. PergamonPress, Oxford, 1987. 539 p. Translate from *Gidrodinamika*, 3rd ed., Moscow, Nauka Publ., 1986, 736 p.
12. *Gorshkov A.V., Prosviryakov E.Y.* Complex stationary convection with third-kind boundary conditions at the boundaries of a fluid layer. *Diagnostics, Resource and Mechanics of materials and structures*, 2016, iss. 2, pp. 34–47. Available at: http://dream-journal.org/issues/2016-2/2016-2_81.html .

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