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ASYMPTOTIC EXPANSION OF A SOLUTION OF A SINGULARLY PERTURBED OPTIMAL CONTROL PROBLEM IN THE SPACE $\mathbb{R}^n$ WITH AN INTEGRAL CONVEX PERFORMANCE INDEX

A. A. Shaburov

We consider an optimal control problem with an integral convex performance index for a linear stationary control system in the class of piecewise continuous controls with a smooth constraint on the control. In the general case, the Pontryagin maximum principle is a necessary and sufficient optimality condition in this problem. We derive an equation for the initial vector of the adjoint system in the general case. Then this equation is adapted to the optimal control problem with an integral convex performance index for a linear system with fast and slow variables. We show that the solution of this equation tends to the solution of the equation corresponding to the limit problem as the small parameter tends to zero. The obtained results are applied to study a problem describing the motion of a material point in $\mathbb{R}^n$ on a fixed time interval. We construct the asymptotics of the initial vector of the adjoint state; this vector defines the form of the optimal control. It is shown that the asymptotics is of power type.

Keywords: optimal control, singularly perturbed problems, asymptotic expansions, small parameter.

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Aleksandr Aleksandrovich Shaburov, doctoral student, Ural Federal University, Yekaterinburg, 620002 Russia, e-mail: alexandershaburov@mail.ru .

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