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STABILITY ITERATIONS AND AN EVASION PROBLEM WITH A CONSTRAINT ON THE NUMBER OF SWITCHINGS

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For an approach–evasion differential game, we consider a variant of the method of program iterations called stability iterations. A connection is established between the iterative procedure and the solution of an evasion problem with a constraint on the number of switchings: the stability iterations define the successful solvability set of the problem. It is proved that the evasion is possible if and only if the strict evasion is possible (i.e., the evasion with respect to neighborhoods of sets defining the approach–evasion game). We specify a representation of the strategies that guarantee the evasion with a constraint on the number of switchings. These strategies are defined as triplets whose elements are a multidimensional positional control strategy, a correction strategy realized as a mapping that takes a game position to a nonanticipating multifunctional on the trajectory space and defines the choice of the switching times, and a positive integer that satisfies the constraints on the number of switchings and specifies the number of switchings of the control. It is important that we use nonanticipating multifunctionals as a tool for generating the controls of the evading player. The paper is in line with the research carried out by N. N. Krasovskii's school on control theory and the theory of differential games.

Keywords: nonanticipating multifunctional, stability operator, correction strategy.

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