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PERTURBATION OF A WAVEGUIDE BY A NARROW POTENTIAL

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We study a boundary value problem in a band with Fourier boundary condition. The problem models a waveguide perturbed by a narrow complex potential of large intensity. The potential depends on a small parameter and a large parameter. The small parameter corresponds to the diameter of the support of the potential, and the large parameter corresponds to the maximum value of the potential. The product of the parameters tends to zero. The problem corresponds to mathematical models of a quantum waveguide and an acoustic waveguide. In this statement, in contrast to the statements considered earlier, a weaker constraint is imposed on the ratio of the parameters and the Fourier conditions are given at the boundary of the band. The main content of this paper is the construction of a special transform that takes the original operator to an operator with a small localized perturbation; the transform preserves the spectrum of the operator. We obtain a condition on the potential under which an eigenvalue appears from the edge of the continuous spectrum; in this case, we find the leading terms of the asymptotics of the eigenvalue. We also obtain a condition for the absence of such an eigenvalue. The results are formulated in a theorem.

Keywords: perturbation, waveguide, eigenvalue.

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