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## ON THE NONTRIVIAL SOLVABILITY OF ONE CLASS OF NONLINEAR INTEGRAL EQUATIONS OF THE URYSOHN TYPE

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This paper is devoted to the study and solution of one class of nonlinear integral equations of the Urysohn type on the positive half-line. Special cases of such equations are applied in various areas of mathematical physics. They appear, in particular, in kinetic gas theory, in radiative transfer theory, and in  $p$ -adic mathematical physics. It is assumed that the Hammerstein convolution operator with a power nonlinearity is a minorant in the Krasnoselskii sense for the Urysohn operator. We prove a theorem of existence of nonnegative nontrivial bounded solutions. In addition, we find the limit of the constructed solution at infinity. The monotonicity of the solution is established in a special case. The proof of the main theorem is of constructive nature. The proof is based on the construction of invariant conical segments for the corresponding nonlinear Urysohn operator. In the end of the paper, we give examples of equations of the described type that are of independent interest.

Keywords: Urysohn equation, Caratheodory condition, Volterra equation, iteration, monotonicity, bounded solution.

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