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ASYMPTOTICS OF A SOLUTION OF A THREE-DIMENSIONAL NONLINEAR WAVE EQUATION NEAR A BUTTERFLY CATASTROPHE POINT

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In the framework of the method of matched asymptotic expansions, a solution of the three-dimensional nonlinear wave equation $-U''_{TT}+U''_{XX}+U''_{YY}+U''_{ZZ} = f(\varepsilon T, \varepsilon X, \varepsilon Y, \varepsilon Z, U)$ is considered. Here ε is a small positive parameter and the right-hand side is a smoothly changing source term of the equation. A formal asymptotic expansion of the solution of the equation is constructed in terms of the inner scale near a typical "butterfly" catastrophe point. It is assumed that there exists a standard outer asymptotic expansion of this solution suitable outside a small neighborhood of the catastrophe point. We study a nonlinear second-order ordinary differential equation (ODE) for the leading term of the inner asymptotic expansion depending on three parameters: $u''_{xx} = u^5 - tu^3 - zu^2 - yu - x$. This equation describes the appearance of a step-like contrast structure near the catastrophe point. We briefly describe the procedure for deriving this ODE. For a bounded set of values of the parameters, we obtain a uniform asymptotics at infinity of a solution of the ODE that satisfies the matching conditions. We use numerical methods to show the possibility of locating a shock layer outside a neighborhood of zero in the inner scale. The integral curves found numerically are presented.

Keywords: matched asymptotic expansions, nonlinear ordinary differential equation, nonlinear equation of mathematical physics, butterfly catastrophe, numerical methods.

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