

DOI: 10.21538/0134-4889-2017-23-2-239-249

MSC: 35J15, 35J25, 35B25, 35B40, 35C20

## THE GENERALIZED BOUNDARY FUNCTION METHOD FOR BISINGULAR PROBLEMS IN A DISK

D. A. Tursunov

The aim of the research is the development of the asymptotic method of boundary functions. This work is devoted to constructing complete asymptotic expansions of the solutions of boundary value problems for a bisingular inhomogeneous linear second-order elliptic equation. The equation has the following singularities: there is a small parameter at the Laplace operator and the corresponding unperturbed (limit) equation has singularities both at the boundary of the disk and inside it. Bisingular Dirichlet, Neumann and Robin problems are studied in the disk. Complete asymptotic expansions of the solutions of the bisingular problems are constructed by the generalized method of boundary functions, which differs from the matching method in that the growing singularities of the outer expansion are actually removed from it and included in the inner expansions with the help of auxiliary asymptotic series. The resulting solutions are asymptotic in the sense of Erdelyi, and the asymptotic expansions are Puiseux series. The leading terms of the asymptotic expansions of the solutions are negative fractional powers of the small parameter. The obtained asymptotic expansions of the solutions of the boundary value problems are justified by means of the maximum principle.

Keywords: asymptotic expansion of the solution, bisingular problem, second-order elliptic equation, modified Bessel functions, Dirichlet problem, Neumann problem, Robin problem, generalized boundary function method, small parameter, maximum principle.

### REFERENCES

1. Il'in A.M. *Matching of asymptotic expansions of solutions of boundary value problems*. Providence, RI: AMS, 1992, Ser. Transl. Math. Monogr., 102, 281 p. ISBN: 978-0-8218-4561-5.
2. Il'in A. M., Danilin, A. R. *Asimptoticheskie metody v analize* [Asymptotic methods in analysis]. Moscow, Fizmatlit Publ., 2009, 248 p. ISBN: 978-5-9221-1056-3.
3. Rozov N.Kh. Some remarks on bisingular boundary-value problems. *J. Math. Sci. (N. Y.)*, 2007, vol. 144, no. 4, pp. 4241–4245. doi: 10.1007/s10958-007-0266-3.
4. Danilin A.R. Asymptotic behaviour of solutions of a singular elliptic system in a rectangle. *Sbornik: Mathematics*, 2003, vol. 194, no. 1, pp. 31–61. doi: 10.1070/SM2003v194n01ABEH000705.
5. Lelikova E.F. On the asymptotics of a solution to an equation with a small parameter in a neighborhood of a point of inflexion. *Tr. Inst. Mat. i Mekh. UrO RAN*, 2016, vol. 22, no. 1, pp. 197–211 (in Russian).
6. Khachai O.Yu. Asymptotic expansion of the solution of the bisingular Cauchy problem for nonlinear first-order ordinary differential equation. *Dep. VINITI*, 2005, vol. 16, no. 174-V2005, pp. 1–46 (in Russian).
7. Tursunov D.A., Erkebaev U.Z. Asymptotic expansions of solutions to Dirichlet problem for elliptic equation with singularities. *Ufa Math. J.*, 2016, vol. 8, no. 1, pp. 97–107. doi: 10.13108/2016-8-1-97.
8. Tursunov D.A. Erkebaev U.Z. Asymptotics of the Dirichlet problem solution for a bisingular perturbed equation in the ring. *Vestnik Udmurtskogo Univ. Mat. Mekh. Komp. Nauki*, 2015, vol. 25, no. 4, pp. 517–525 (in Russian).

9. Oleinik O.A. On properties of solutions of certain boundary problems for equations of elliptic type. *Mat. Sbornik*, 1952, vol. 30(72), pp. 695–702 (in Russian).
10. Fedoryuk M.V. *Asymptotic analysis: linear ordinary differential equations*. Berlin, Springer-Verlag, 1993, 363 p. doi: 10.1007/978-3-642-58016-1. Original Russian text published in *Asimptoticheskie metody dlja linejnyh obyknovennyh differencial'nyh uravnenij*, Moscow, Nauka Publ., 1983, 352 p.

The paper was received by the Editorial Office on June 9, 2016.

*Dilmurat Abdillazhanovich Tursunov*, Dr. Phys.-Math. Sci., Prof., Osh State University, Osh, 723500 Kyrgyzstan, e-mail: tdaosh@gmail.com

Cite this article as:

D. A. Tursunov, The generalized boundary function method for bisingular problems in a disk, *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 2, pp. 239–249.