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A BOUND FOR THE REMAINDER TERM IN THE ASYMPTOTIC EXPANSION OF THE ELLIPTIC SINE CONTAINING THE FIRST THREE TERMS

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We propose a simple method for finding an asymptotic expansion of the elliptic sine $z = \operatorname{sn}(u; k)$ in powers of $k^2 - 1$. In the literature only the first two terms of the expansion have been written. The proposed method makes it possible to find subsequent terms of the expansion. The disadvantage of this method is its computational intensity. We prove that the remainder term $R(u, k)$ in the asymptotic expansion containing the first three terms of the expansion satisfies the limit equality

$$\lim_{z \rightarrow 1} \lim_{k \rightarrow 1} R(u, k) \frac{(1-z)^2}{(1-k^2)^3} \neq 0.$$

The main result of this paper is an estimate for the remainder term. We prove that

$$|R(u, k)| \leq \operatorname{const} \frac{1}{\cosh^2 u} \frac{(1-k^2)^3}{(1-z)^3}.$$

Keywords: elliptic sine, asymptotic expansions, hyperbolic functions.

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